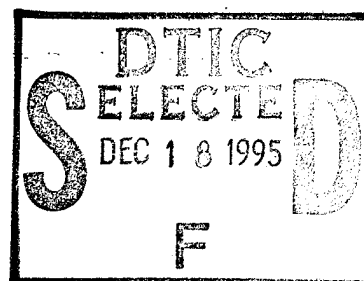


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New Theoretical Approach to the Strength of  
Fibre Composite Hybrid Materials

Plymouth Polytechnic (England)



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New Theoretical Approach to the Strength of Fibre Composite Hybrid Materials,

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by D. Short, and J. Summerscales.

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A fiber composite hybrid material generally consists of two or more fiber types contained in a single matrix. There are several equations already in existence which allow the calculation of the strength of the resulting composite. However, the most commonly used equations assume either a sharp transition in the failure strain of the composite when the volume ratio of the two fiber species crosses a critical value of volume ratio, or that there are distinct failure strains for each of the two fiber species. In the new theory it is assumed that the failure strain of the low elongation component can be linearly increased to the failure of the high elongation component in a manner predicted by the rule-of-mixtures. The equation predicts an upper limit to strength which may show the 'hybrid effect', -a synergistic strengthening of the composite properties, subject to the correct choice of constituents for the hybrid.

KEYWORDS: \*Fiber composites, \*Foreign technology.

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"A new theoretical approach to the strength of fibre composite hybrid materials"

Presented at the 13th BPF Reinforced Plastics Congress,  
Brighton, November 1982, Paper 53, Pages 225-229.

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#### ABSTRACT:

A fibre composite hybrid material generally consists of two or more fibre types contained in a single matrix. There are several equations already in existence which allow the calculation of the strength of the resulting composite. However, the most commonly used equations assume either a sharp transition in the failure strain of the composite when the volume ratio of the two fibre species crosses a critical value of volume ratio, or that there are distinct failure strains for each of the two fibre species. In the new theory it is assumed that the failure strain of the low elongation component can be linearly increased to the failure of the high elongation component in a manner predicted by the rule-of-mixtures. The equation predicts an upper limit to strength which may show the "hybrid effect", a synergistic strengthening of the composite properties, subject to the correct choice of constituents for the hybrid.

#### 1: INTRODUCTION

In an aligned single-fibre composite the modulus of the composite in the direction of the fibre,  $E_c$ , can be calculated from the volume fraction of each component,  $v_x$ , and the modulus of that component,  $E_x$ . If the subscripts f and m indicate fibre and matrix respectively, then the initial

composite modulus in the direction of the fibre will be given by equation (1):

$$E_c = v_f E_f + v_m E_m = v_f E_f + (1 - v_f) E_m \quad (1)$$

because the sum of the volume fractions must be unity, as in equation (2):

$$v_f + v_m = 1 \quad (2)$$

In an aligned two-fibre composite, commonly known as a hybrid composite, then equations (1) and (2) can be expanded:

$$E_c = v_a E_a + v_b E_b + v_m E_m = v_a E_a + v_b E_b + (1 - v_a - v_b) E_m \quad (3)$$

$$v_a + v_b + v_m = 1 \quad (4)$$

where a and b indicate each of the fibre species. In practice, the contribution of the matrix to the overall stiffness of the composite is negligible, and is normally ignored:

$$E_c = v_a E_a + v_b E_b \quad (5)$$

An alternative method of calculating the initial modulus is to consider the hybrid as an equivalent material fabricated from two composites. If the ratio of the volume fraction of one composite component,  $V_x$ , to the total volume of composite is denoted  $R_x$ , then:

$$E_A = v_a E_a + v_m E_m ; E_B = v_b E_b + v_m E_m \quad (6) \quad (7)$$

$$R_A = V_A / (V_A + V_B) ; R_B = V_B / (V_A + V_B) \quad (8) \quad (9)$$

$$E_{ci} = R_A E_A + R_B E_B ; R_A + R_B = 1 \quad (10) \quad (11)$$

$$E_{ci} = v_a E_a + v_b E_b + v_m E_m \quad (12)$$

If both the fibres have brittle properties, then the modulus of the composite after failure of all the low elongation component (A) will be given by:

$$E_{cii} = R_B E_B = v_b E_b + v_m E_m \quad (13)$$

The above equations assume that the matrix remains intact until failure of the composite, and that the fibre failure strain distributions do not overlap. The failed fibres are now assumed to be either completely debonded, or broken into sections which are shorter than the "critical length", so that they now make no significant contribution to the modulus. However, the presence of the fractured ends of these fibres is also assumed to cause no stress concentration on the adjacent high elongation fibres (species B). Equation (12) is now generally accepted as the "rule of mixtures" for the initial elastic modulus of a hybrid composite.

## 2.1: STRENGTH : Case One (2)

In addition to the above equations Hayashi (1) derived expressions

for the tensile strength of a brittle two-fibre elastic composite. Until the failure of the low elongation component, the tensile stress and the strength of the hybrid at this failure, will be given by:

$$\sigma_i = E_i \epsilon \quad \sigma_i^1 = E_i \epsilon_a^1 \quad (14)$$

where  $\epsilon_a^1$  is the failure strain of the low elongation fibre.

At the failure of the low elongation component there will be a discontinuity, or a gradual change in the slope, in the stress-strain curve, and until the final failure:

$$\sigma_{ii} = E_{ii} \epsilon \quad \sigma_{ii}^1 = E_{ii} \epsilon_b^1 \quad (15)$$

where  $\epsilon_b^1$  is the failure strain of the high elongation fibre. This assumes that all the fibres of a single type fail at a definite strain, and then no longer contribute to the load carrying capability of the composite. The work of fraction (i.e. the total elastic strain energy) up to complete separation will be given by:

$$U = \frac{1}{2} E_A R_A (\epsilon_a^1)^2 + \frac{1}{2} E_B R_B (\epsilon_b^1)^2 \quad (16)$$

In the case where the high elongation component contributes to the load carrying capability of the hybrid throughout the range of compositions the equations (17) and (18) will apply. The strength of the hybrid will be given by the equation which predicts the greatest value of stress:

$$\sigma^1 = (R_L E_L + R_H E_H) \epsilon_L^1 \quad (17)$$

$$\sigma^1 = R_H E_H \epsilon_H^1 \quad (18)$$

The minimum strength of the hybrid will occur at a specific volume fraction which can be derived by equating the two strength equations such that:

$$(R_L E_L + R_H E_H) \epsilon_L^1 = R_H E_H \epsilon_H^1 \quad (19)$$

which can be rearranged, with  $R_H = 1 - R_L$ , to give:

$$\frac{1 - R_L}{R_L} = \frac{E_L \epsilon_L^1}{E_H (\epsilon_H^1 - \epsilon_L^1)} \quad (20)$$

and hence:

$$\frac{1}{R_H} = 1 + \frac{E_H}{E_L} \left[ \frac{\epsilon_H^1}{\epsilon_L^1} - 1 \right] \quad (21)$$

which allows the determination of the point T in figure 1.

In this analysis there will be a critical volume content of the high strain fibres (low modulus fibres) above which the fracture mode changes from being dominated by the low elongation fibre, to domination by these high elongation fibres. An identical expression for this critical ratio, expressed in terms of the failure stress of the fibres, has been derived by both Gunyaev<sup>(3)</sup> and Chamis<sup>(4)</sup>:



$$V_{CRIT} = \left[ 1 + \frac{\sigma_H^1}{\sigma_L^1} - \frac{E_H}{E_L} \right]^{-1} \quad (22)$$

The failure strength of this minimum strength hybrid associated with the critical content is given by:

$$\sigma_{min}^1 = (1 - V_{CRIT})E_L \epsilon_L^1 \quad (23)$$

A characteristic inflection will occur in the stress-strain curve corresponding to the limiting elongation of the low strain fibres.

## 2.2: STRENGTH : Case Two

In order to estimate the tensile strength of two-fibre hybrids it is tempting to use the rule of mixtures for hybrid strength:

$$\sigma^1 = R_L \sigma_L^1 + R_H \sigma_H^1 \quad (24)$$

However, if the individual components are linear elastic materials until failure, such that  $\sigma_a^1 = E_a \epsilon_L^1$ , then equation (24) can be rearranged to give:

$$\sigma^1 = R_L E_L \epsilon_L^1 + R_H E_H \epsilon_H^1 \quad (25)$$

which requires that the individual components of any hybrid composite will fail simultaneously, but at different strains. This is obviously not a

very satisfactory representation of the truth.

### 3: MODEL AND ASSUMPTIONS

In the theoretical model which follows the basic assumptions are that the failure strain of the hybrid composite will follow the rule of mixtures:

$$\epsilon^1 = v_L \epsilon_L^1 + v_H \epsilon_H^1 \quad (26)$$

and that the first failure of the fibres will occur at random throughout the complete volume of the composite.

In a hybrid composite in which the brittle low elongation fibres are isolated from one another because of the intimacy with which the two fibre species are interspersed, it is possible that the catastrophic crack propagation which would occur in the brittle monofibre composite can be suppressed by the presence of the tough fibres. In an optimised mixture this could allow individual brittle fibres to fail at their own weakest points because they would only rarely be subjected to the dynamic stress concentration due to fracture of an adjacent brittle fibre. The strain energy released would therefore be absorbed by the tough fibres, to result in a small local overstraining of the composite. Provided that the failed brittle fibres are uniformly distributed throughout the composite, and that the frequency of fracture within any single fibre is such that the unbroken lengths are greater than the critical length, it is probable that these brittle fibres will continue to carry load and hence contribute to

the hybrid modulus and strength.

Parratt (5) has reported that 25% HMS-carbon and 75% R-glass hybrid composites show yielding behaviour as the carbon fibres break up. Although fracture starts very consistently at a strain characteristic of the bundle strength of HMS-carbon fibres, these fractured fibres continue to carry load and are broken down to a measured length of 0.8 mm, requiring that three or four fractures must occur in each fibre to accommodate the final composite breaking strain of 1.9%.

Manders (6) has reported failure strain enhancement of up to 35% in high tensile strength carbon fibres when incorporated into a hybrid composite with glass fibres, and enhancements of up to 46% in high modulus carbonfibre-glassfibre hybrids.

Based on the expectation that the failure strain of hybrid composites will follow the rule-of-mixtures for fibres with similar elongations to failure we now propose to derive a new expression for the upper bound of hybrid strength. The increase in strength due to the enhancement of failure in the low elongation fibre,  $\sigma^{\Delta}$ , will thus be given by the expression:

$$\sigma^{\Delta} = V_L E_L (1 - V_L) (\epsilon_H^1 - \epsilon_L^1) \quad (27)$$

and similarly the degradation of strength required in order that the composite fails at a constant strain,  $\sigma^V$ , will be given by:

$$\sigma^V = V_H E_H (1 - V_H) (\epsilon_L^1 - \epsilon_H^1) \quad (28)$$

The factor for strength degradation is of course negative because by definition  $\epsilon_H^1 > \epsilon_L^1$ , and equation 28 can thus be rewritten:

$$\sigma^V = -V_H E_H (1 - V_H) (\epsilon_H^1 - \epsilon_L^1) \quad (29)$$

The total strength of the hybrid composite will therefore be given by the two terms in the rule-of-mixtures strength expression (equations 24 and 25) and the two expressions for a change to a rule-of-mixtures failure strain (equations 27 and 29) to give:

$$\sigma^1 = V_L E_L \epsilon_L^1 + V_H E_H \epsilon_H^1 + V_L E_L (1 - V_L) (\epsilon_H^1 - \epsilon_L^1) - V_H E_H (1 - V_H) (\epsilon_H^1 - \epsilon_L^1) \quad (30)$$

which can be simplified by rearranging and taking  $(1 - V_L) = V_H$  to give:

$$\sigma^1 = V_L E_L \epsilon_L^1 + V_H E_H \epsilon_H^1 + V_L V_H (E_L - E_H) (\epsilon_H^1 - \epsilon_L^1) \quad (31)$$

The third term of this expression will therefore predict the magnitude of any benefit to be expected in the composite strength through the judicious combination of different fibres. Obviously, when the moduli of the two component composites are equal (figure 1), or when the failure strains of the component composites are equal (figure 2), there will be no benefit to the hybrid strength. Maximum benefit is therefore to be expected by maximising the difference in either (or both) the failure strains or (and) the elastic moduli of the components (figure 3).

However, if the difference in failure strains is too great then the assumptions of the model will be broken (figures 4 and 5), as the low strain fibres will be multi-fractured, and the interaction of these defects will affect the strength because of the increased stress concentrations on the unfractured fibres.

This theory takes no account of the benefits which may arise because of the thermal residual strains due to the differences in the coefficients of thermal expansion between the two fibres. These will be evenly spread throughout the intimate composite rather than concentrated at the laminate interfaces. As an example of the beneficial effect this can lead to, we shall consider a carbon fibre and glass fibre hybrid reinforced plastic. Carbon fibres are unusual in that they undergo a slight longitudinal expansion upon cooling. It seems reasonable that when constrained by a contracting gfrp matrix during cure and cooling, this could lead to a net compressive strain in the fibre at no net elongation of the composite, with the consequent benefits in tensile loading.

#### 4: EXPERIMENTAL PROCEDURE

Unidirectional fibre composites were fabricated by a leaky-mould compression moulding technique. The carbon fibres were Courtaulds Type EAS 10K filament tows (batch No. E3A 105C, and No. 144R). The glass fibres were 'Type XRE 23 47', 2400 filament tex (product WX 60 41, reference 18 120 070 012). The resin was Scott Bader Crystic 272 cured with 1% catalyst M and 1% accelerator E. The composites were cured at

room temperature for 24 hours to minimise thermal strains, followed by postcure at 70°C. The composites were manufactured in moulds of 2 mm. x 5 mm. cross section. Ten tows of carbon fibre or 10 tex of glass fibres were used in each monofibre composite.

The hybrid composites consisted of five tows of the carbon fibres and five tex of the glass fibres. Two levels of microstructure could be defined: coarse and fine. The coarse hybrids consisted of composites fabricated with fibres as supplied. The fine hybrids used fibres which had been separated and intermingled using a modification of the Courtaulds Air Knife.

Three point flexural testing with a span of 80 mm. and a crosshead speed of 5 mm/min on an Instron TT-DM machine. Failure of the glass fibre reinforced plastics was by progressive delamination directly beneath the loading anvil. Failure of the carbon fibres reinforced plastics was by catastrophic crack propagation from the tensile face, although in some cases total separation of the two halves did not occur. Failure of the carbon/glass hybrids was by local buckling of fibres out of the compressive surface of the beam.

Because the hybrid failure mode is generally neither by delamination nor by tensile crack propagation, but by a third 'buckling' mode, the strengths of the hybrid composites do not generally rise to the values predicted by the theory. As a result of this buckling failure mode the neutral axis of the beam will move towards the tensile face of the beam and earlier failure is therefore to be expected.

In Figures 6 and 7 the experimental values of strength for hybrids with both coarse (+) and fine (V) mixing of the two fibre species are presented. In Figure 6 the lines represent strengths predicted from the average elastic moduli and failure strains of the two monofibre composites. In Figure 7 the lines represent strengths predicted from the elastic moduli and failure strains of the poorest monofibre beams. As the 'hybrid effect' is predicted by the term  $+ V_L V_H (E_L - E_H)(\epsilon'_H - \epsilon'_L)$  and  $E_X$  and  $\epsilon_X$  will be constant for any combination of specific composites, the maximum will occur when  $V_L V_H = 0.25$ , that is when 50% of each composite is included. Because the predicted 'hybrid effect' will be greatest with equal proportions of each of the component composites, all experimental hybrid results were obtained using composites with half of the number of filament tows in each of the monofibre composites. In the interests of clarity the fine and coarse hybrids have been moved slightly away from 50% volume percentage line in the graphs.

Figure 8 shows the strengths of the composites plotted against the moduli of the same composites. The symbols used are

- x : monofibre glass composite
- : monofibre carbon fibre composite
- V : fine mix hybrid composite
- +

The rule-of-mixture lines for average elastic moduli (AA), average elastic strengths (BB) and worst - case elastic strengths (CC) have been added.

## 5: CONCLUSIONS

A new equation for the prediction of the strength of hybrid composites is presented, which can be used to indicate the extent of any "hybrid effect". It is assumed that the failure strain of the low elongation component can be linearly increased to the failure strain of the high elongation composite, in line with the strain predicted by the rule-of-mixtures, provided that the difference in these two strains is not extreme. The two species of fibres should be so intimately mixed that any stress concentration from a broken fibre is sufficiently isolated by the surrounding high elongation fibres that the energy released from the broken fibres is absorbed by the matrix and high elongation fibres without fracture, to prevent consecutive failure of adjacent low strain fibres, and hence eliminate the catastrophic failure which is associated with monolithic brittle fibre composites.

## ACKNOWLEDGEMENTS

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## REFERENCES

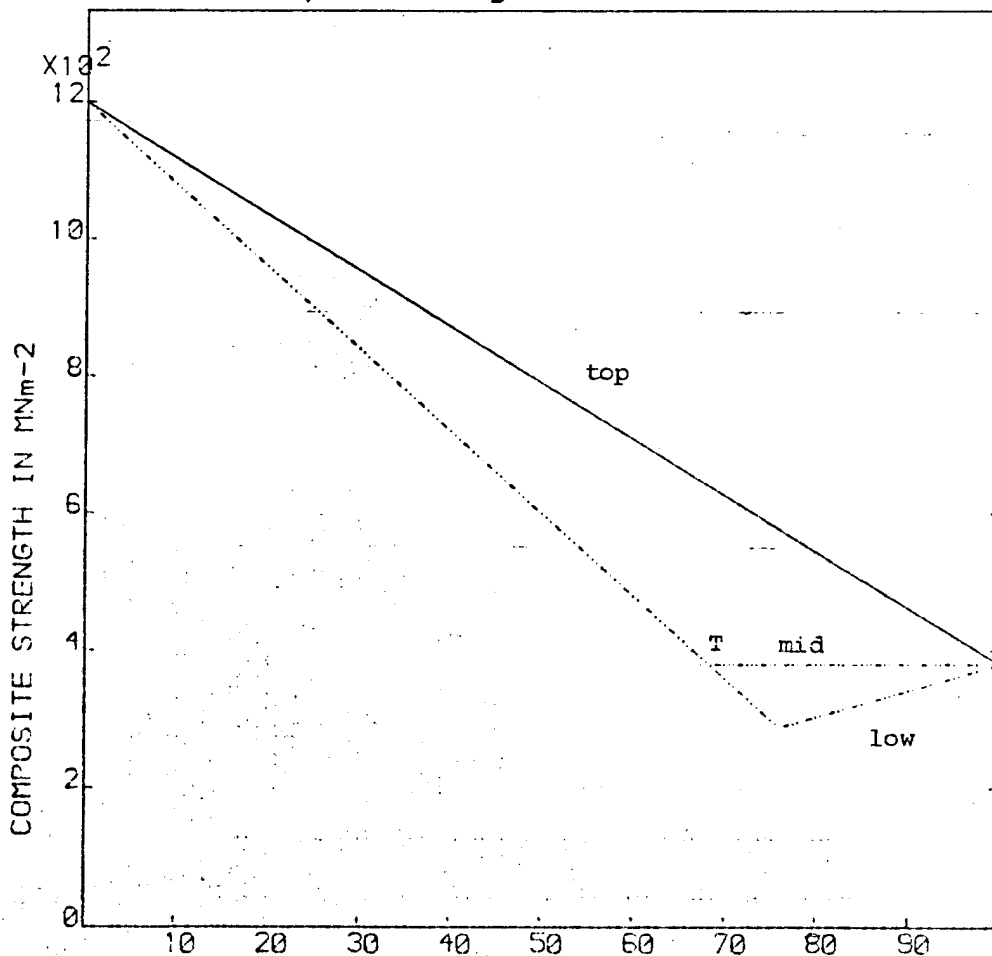
- (1) T. Hayashi, "On the improvement of mechanical properties of composites by hybrid composition", 8th Reinforced Plastics Group Congress (British Plastics Federation, Brighton, 1972). Paper 22, pp. 149 - 152.
- (2) N.J. Parratt and K.D. Potter, "Behaviour and benefits of hybrid reinforced composites", PERME Technical Report TR 81, November 1978.
- (3) G.M. Gunyaev, "Polycomponent high modulus composites", Polymer Mechanics 13(5), September-October 1977, pp. 685 - 692 (translation from the Russian).
- (4) C.C. Chamis and J.H. Sinclair, "Prediction of properties of intraply hybrid composites", NASA Technical Memorandum 79087 and "Approximate design - analysis procedures for hybrid composites", 34th Annual Reinforced Plastics/Composites Conference (Society of the Plastics Industry, New Orleans, 1979).
- (5) N.J. Parratt and K.D. Potter, "Mechanical behaviour of intimately-mixed hybrid composites", 3rd International Conference on Composite Materials (Metallurgical Society of AIME et al., Paris, August 1980).
- (6) P.W. Manders, "The strength of mixed fibre composites", Doctoral Thesis, University of Surrey, September 1979.

FIGURE 1. Hybrid of two composites with equal magnitude moduli exhibit rule-of-mixtures strength for hybrid effect curve.

The two monofibre composites are:-

	Steel 40 Vol. % Fibre	carbon I (HM) 20 Vol.% Fibre
Strength (GPa)	1.200	0.380
Modulus (GPa)	80.000	80.000
Elongation	0.015	0.005

TOP LINE: Fibres both fail at r-o-m strain  
MID LINE: HEC fail at strain of majority fibres  
LOW LINE: Component only effective on one side

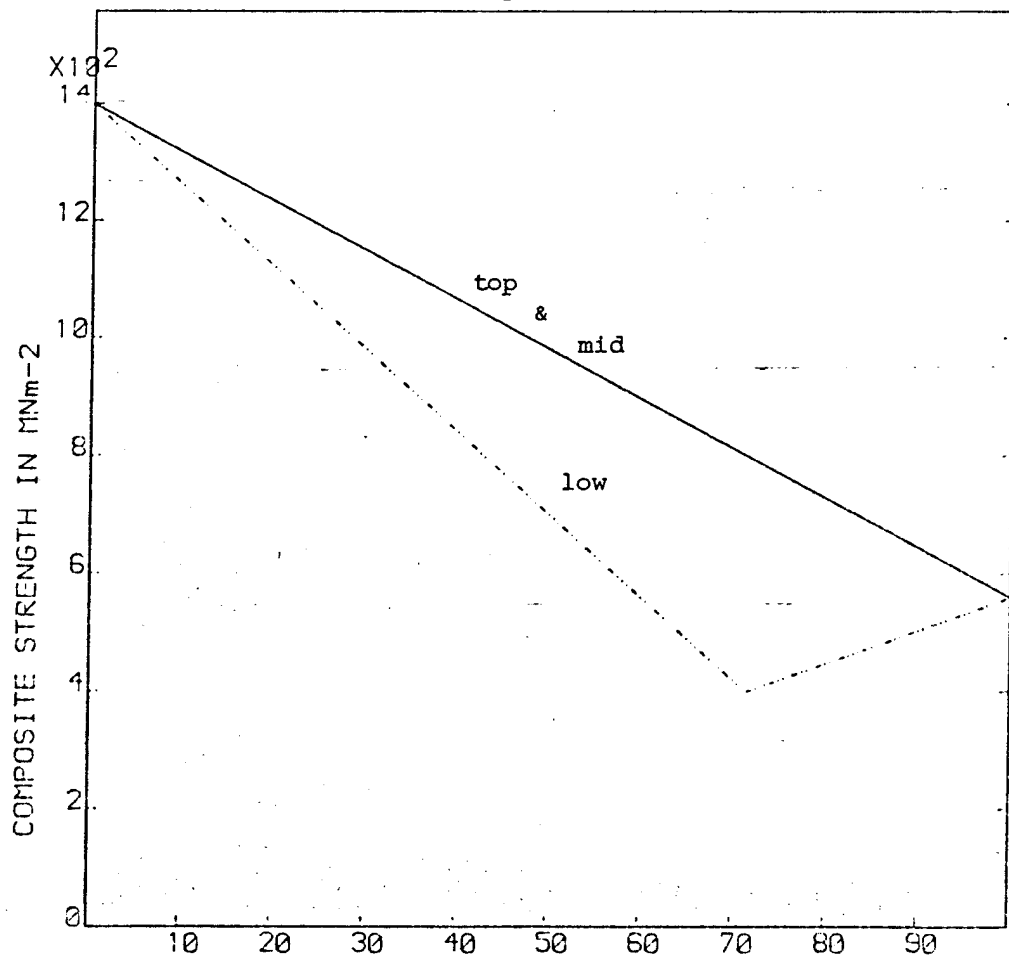


**FIGURE 2:** Hybrid of two composites with equal magnitude failure strains exhibits rule-of-mixture strength for hybrid effect curve.

The two monofibre composites are:-

	Carbon II (HT) 50 Vol.% Fibre	Carbon II (HT) 20 Vol.% fibre
Strength (GPa)	1.300	0.520
Modulus (GPa)	103.500	41.400
Elongation	0.013	0.013

TOP LINE: Fibres both fail at r-c-m strain  
MID LINE: HEC Fail at strain of majority fibres  
LOW LINE: Component only effective on one side

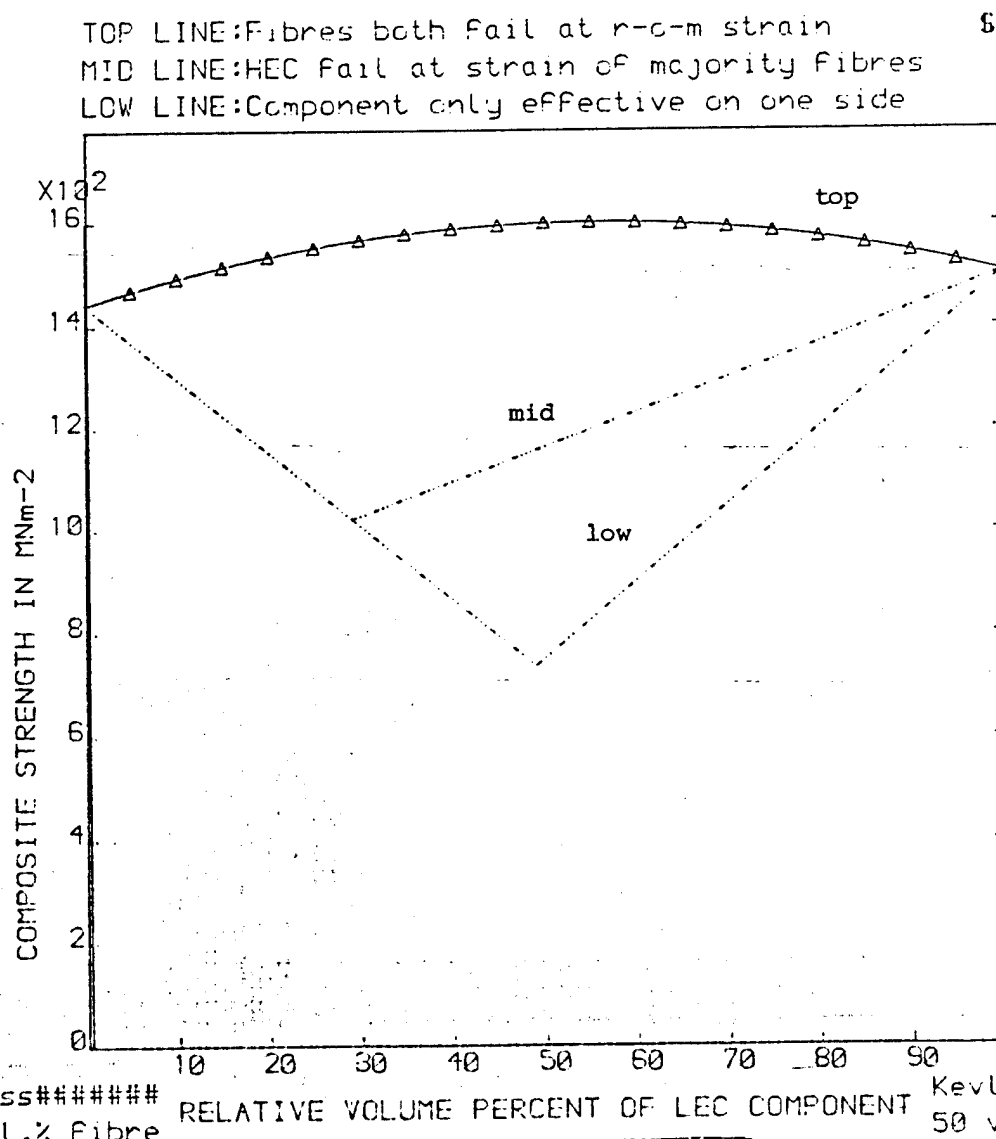


Carbon II (HT) 50 vol.% Fibre      RELATIVE VOLUME PERCENT OF LEC COMPONENT      Carbon II (HT) 20 vol.% Fibre

**FIGURE 3:** Similar moduli and failure strains leading to a small hybrid effect. 17

The two monofibre composites are:-

	E-Glass 50 Vol.% Fibre	Kevlar 49 50 Vol.% Fibre
Strength (GPa)	1.325	1.380
Modulus (GPa)	36.000	65.500
Elongation	0.037	0.021



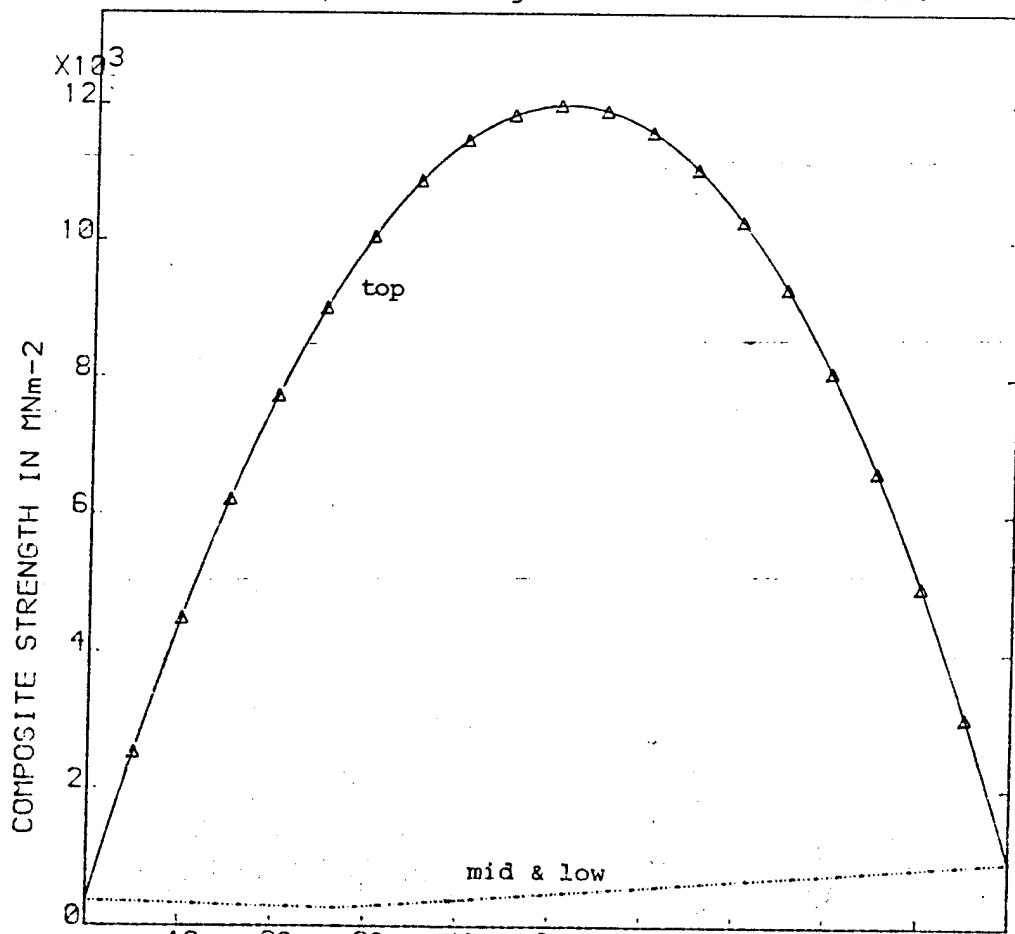
**FIGURE 4:**

Extreme case where carbon I ( $\epsilon^1 = 0.48\%$ ) is expected to carry load at failure strain of nylon ( $\epsilon^1 = 23.3\%$ ). In consequence the predicted hybrid effect is very unlikely to be realised in practice.

The two monofibre composites are:-

	Nylon 50 Vol.% Fibre	Carbon I (HM) 50 Vol.% Fibre
Strength (GPa)	0.350	0.950
Modulus (GPa)	1.500	200.000
Elongation	0.233	0.005

TOP LINE: Fibres both fail at r-o-m strain  
MID LINE: HEC Fail at strain of majority Fibres  
LOW LINE: Component only effective on one side



Nylon#####  
50 vol.% Fibre

RELATIVE VOLUME PERCENT OF LEC COMPONENT

Carbon I (HM)#  
50 vol.% Fibre



FIGURE 5:

Detail of the lower lines shown in Figure 4.

-19-

TOP LINE: Traditional rule of mixtures for strength  
 MID LINE: HEC fail at strain of majority fibres  
 LOW LINE: Component only effective on one side

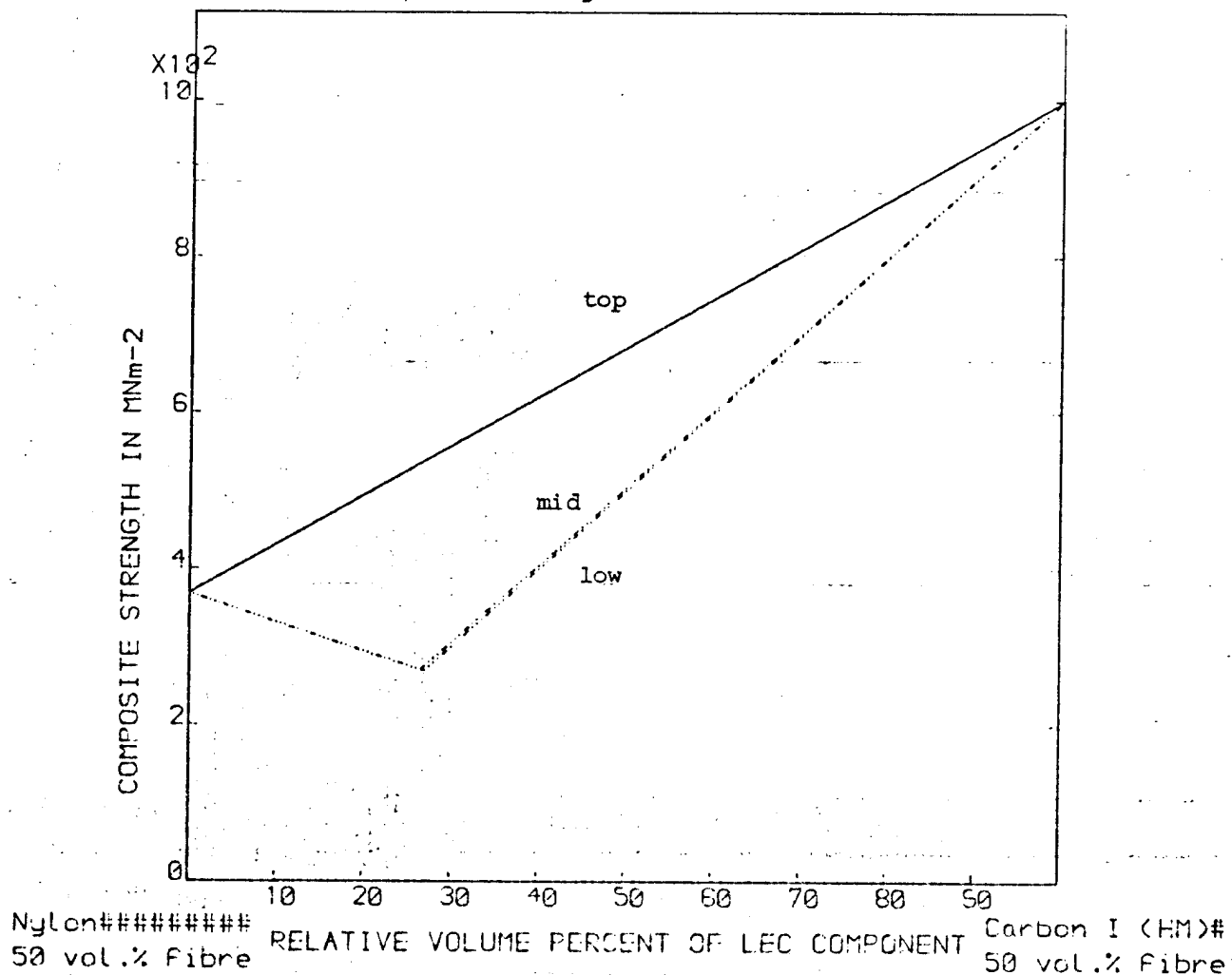


FIGURE 6:

# HYBRID STRENGTH THEORIES vs EXPERIMENTAL DATA: EAS

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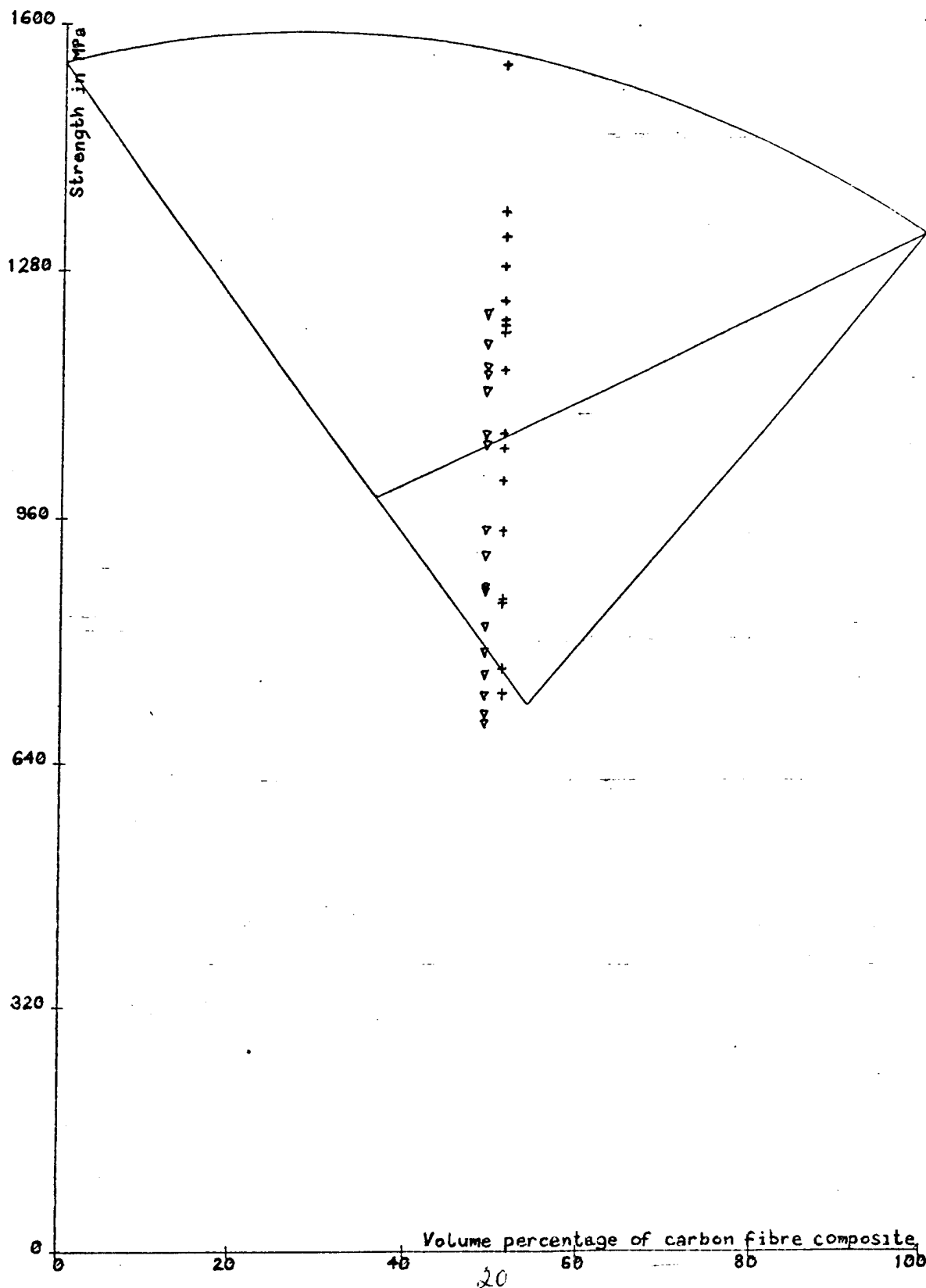


FIGURE 7:

# HYBRID STRENGTH THEORIES vs EXPERIMENTAL DATA: EAS

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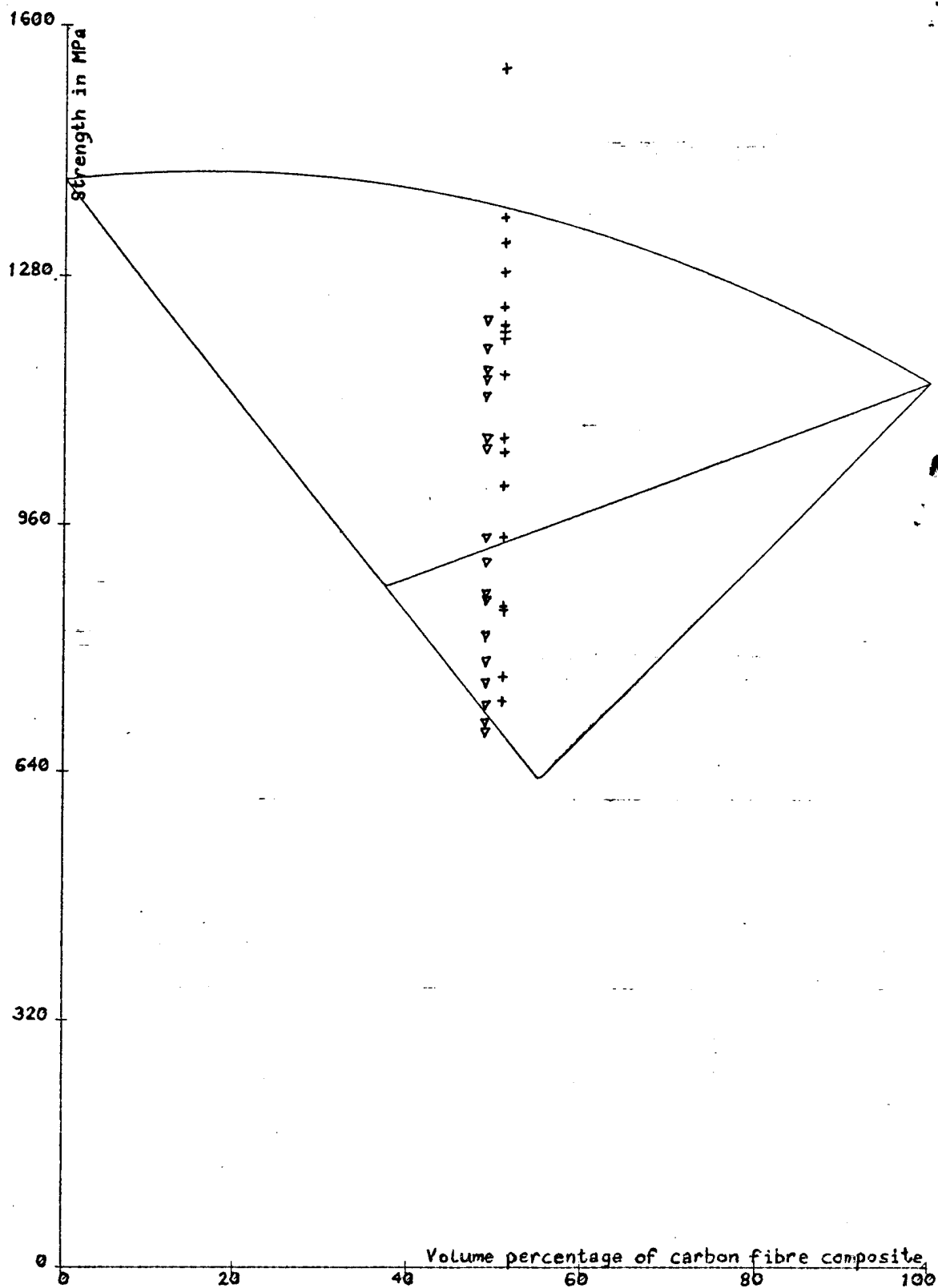




FIGURE 8:

# INITIAL STRENGTH vs ELASTIC MODULUS

EAS

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